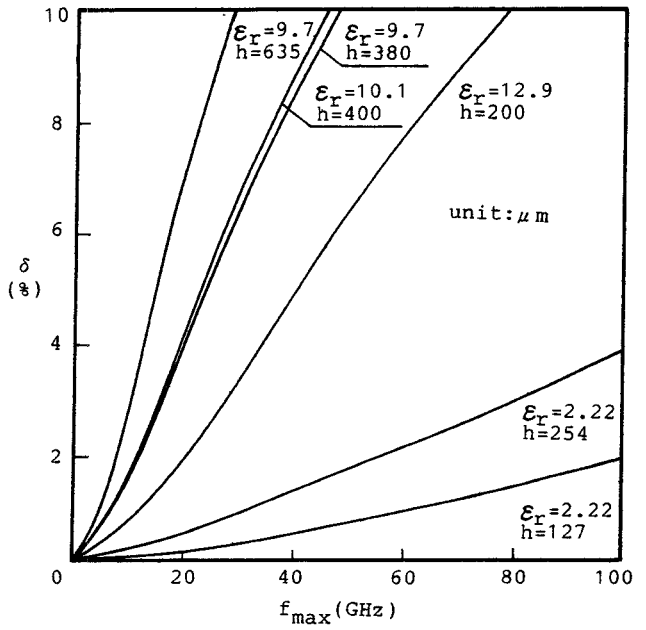

Fig. 2. The deviation factor of guide wavelength, δ, versus the maximum usable frequency, f_{max}.

the guide wavelength obtained with TEM-wave approximation, λ_{TEM}, from the actual guide wavelength, λ. Then the maximum usable frequency, f_{max}, for a given value of δ can be expressed as follows:

$$f_{max} = \frac{c \cdot \eta}{4h\sqrt{\epsilon_r - 1}\left\{0.5 + \left[1 + 2\log_{10}(1 + w/h)\right]^2\right\}} \tag{5}$$

where

$$\eta = \left(\frac{1-\delta}{4\delta}\sqrt{\epsilon_r} \cdot \frac{\lambda_{TEM}}{\lambda_0} - \frac{1}{4\delta}\right)^{-0.667} \tag{6}$$

and

$$\delta = \frac{\lambda_{TEM} - \lambda}{\lambda_{TEM}} \tag{7}$$

and c is the velocity of light in vacuum.

The relationships between the tolerable deviation factor, δ, and the maximum usable frequency, f_{max}, are shown in Fig. 2. This frequency can be regarded as an upper limit, under which the approximate formulas presented in this paper can be used within a specific degree of tolerance for dispersion.

V. Conclusion

Design parameters of micro–coplanar striplines on a variety of substrate materials have been calculated with the rectangular boundary division method. After a least-square curve-fitting procedure, a group of approximate formulas has been obtained for practical use in the design of this new type of transmission line. Experimental data were presented for comparison with theoretical results.

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Reflection of Electromagnetic Waves from Rough Waveguides

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***Abstract*—The reflection coefficient of a section of randomly rough waveguide is calculated by using a coordinate transformation developed by Mallick and Sanyal. We perform a perturbation analysis, assuming that the amplitude of the roughness is small compared to the average width of the waveguide. A drastic difference at long wavelengths between TEM on the one hand and TE and TM on the other has been found.**

I. Introduction

The effects of surface roughness on the propagation of electromagnetic waves is important in the fields of precision microwave measurements and standards [1]. Roughness also partly determines the properties of the waveguides and the Q factor of resonant cavities [2]. Previous work on rough waveguides has focused on the limit where the width of the waveguide varies slowly along its length [3], [4]. Much work has also been performed on periodic corrugation of waveguides [1], [5] and on scattering from random surfaces [6]. Physically the problem has some similarity to the propagation of elastic waves (phonons) down narrow wires [7].

In this paper we will calculate the effects of roughness on the reflection coefficient of a section of waveguide. Decomposition into modes is achieved by utilizing a coordinate transformation analogous to that developed by Mallick and Sanyal [5]. The significance of the coordinate transformation is that the bounding, rough surface becomes a surface of constant coordinate, facilitating the application of the boundary conditions. The problem reduces to a one-dimensional form, where we may use a perturbation approach to deduce the reflection coefficient.

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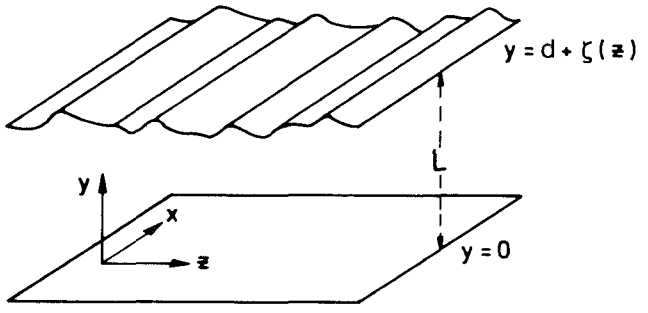

Fig. 1 The upper ($y = d + \zeta(z)$) and lower ($y = 0$) surfaces of the waveguide and the orientation of the coordinate axes.

II. Analysis

We treat a waveguide defined by two perfectly conducting planes, which are separated on average by a region of free space of thickness d. The simplest form of roughness is for only one plane to be affected and, moreover, for the roughness to depend on only one coordinate parallel (*perpendicular* roughness is trivial —it merely defines a new set of modes) to the waveguide. The geometry is shown in Fig. 1. The conducting planes have the y axis as their normal: the rough plane is the surface $y = d + \zeta(z)$, where d denotes the average position and $\zeta(z)$ denotes the fluctuation in the surface due to the roughness; the flat surface is $y = 0$. We assume that $\zeta(z) = 0$ for $z < 0$ and $z > L$, so that we can inject a well-defined wave into the rough guide.

The application of the boundary conditions at the rough surface is clearly problematic in Cartesian coordinates. An orthogonal coordinate system with the surface normals parallel to the (local) basis vectors was constructed by Mallick and Sanyal [5]:

$$u_1 = x$$

$$u_2 = \frac{y}{1+\zeta(z)/d}$$

$$u_3 = z + \Delta(y, z). \tag{1}$$

Here the function $\Delta(y, z)$ is determined by the requirement of orthogonality of the new coordinates, given the definitions of u_1 and u_2. Since we may assume that the roughness varies on a length scale that is large compared with the amplitude of the roughness, we find (by analogy with Mallick and Sanyal [5])

$$\Delta(y, u_3) \simeq \frac{1}{2}\frac{u_2^2}{d}\zeta'(u_3)\left[1 + \frac{\zeta(u_3)}{d}\right]. \tag{2}$$

The full vector wave equation decomposes into two scalar wave equations, whose solutions, ψ, generate the various components of the fields (as long as we have the anisotropic roughness described above and the appropriate initial conditions). So let us consider a single scalar wave equation. Again following Mallick and Sanyal [5], we see that in the limit of small roughness the wave equation is separable: we may write $\psi(u_1, u_2, u_3) = U_1(u_1)U_2(u_2)U_3(u_3)$.

Given the lack of an x (u_1) dependence in the functional form of the roughness, $\zeta(u_3)$, and assuming that the waveguide is fed in a u_1-independent manner, there will be no u_1 dependence in the solutions. Then the equations for the other two functions of separation, U_2 and U_3, are

$$\frac{d^2U_2}{du_2^2} + k_2^2 U_2 = 0 \tag{3a}$$

$$\frac{d^2U_3}{du_3^2} + Q(u_3)\frac{dU_3}{du_3} + \left[k_0^2 - \frac{k_2^2}{\left(1+\zeta(u_3)/d\right)^2}\right]U_3 = 0 \tag{3b}$$

where $k_0^2 = \omega^2\epsilon\mu$ and

$$Q(u_3) = \frac{1}{d}\frac{\zeta'(u_3)}{1+\zeta(u_3)/d}. \tag{4}$$

We see that the disorder enters into (3b) in two places: in the first-order term and in the denominator of the term involving the eigenvalue associated with U_3. We may readily eliminate the first term, resulting in a form closer to that of a conventional wave equation, using the following transformation:

$$V(u_3) = U_3(u_3)\exp\left\{-\frac{1}{2}\int_0^{u_3} Q(v)\,dv\right\}$$
$$= U_3(u_3)\left(1+\zeta(u_3)/d\right)^{-1/2} \tag{5}$$

where, for convenience, we have chosen the lower limit of the integral to be 0, i.e., the point where the roughness commences. We have used the fact that (4) reveals $Q(u_3)$ to be a perfect derivative. Thus the final form of (3b) is

$$V''(u_3) + \left[\left(k_0^2 - \frac{k_2^2}{\left(1+\zeta(u_3)/d\right)^2}\right) - (1/2)Q'(u_3) - (1/4)Q^2(u_3)\right]V(u_3) = 0. \tag{6}$$

The boundary conditions (on the two planes) determine k_2, yielding a one-dimensional wave equation. The effect of the roughness is to make the propagation constant vary with u_3.

Let us return to discuss the reduction to two scalar wave equations, which would be associated with the TE and TM modes in the absence of roughness. In the presence of roughness it is no longer possible, *in general*, to solve in terms of TE and TM to z modes *the direction of propagation*. However it is still possible to define TE and TM to x modes for the type of (anisotropic) roughness considered here. Moreover if the initial condition (associated with the way in which the waveguide is fed) is independent of x, then we may still use either. In that case the TE to z modes are equivalent to the TM to x modes, and the TM to z modes are equivalent to the TE to x modes. This may be seen as a consequence of the independence of the solutions with respect to u_1 in the equations for the field components [5].

Then the relations between ψ and the field components for the TM to z modes are

$$E_1 = H_2 = H_3 = 0 \qquad H_1 = -i\omega\epsilon\psi_M$$
$$E_2 = -\frac{\partial\psi_M}{\partial u_3} \qquad E_3 = \frac{1}{1+\zeta(u_3)/d}\frac{\partial\psi_M}{\partial u_2} \tag{7}$$

and for the TE to z modes we find

$$H_1 = E_2 = E_3 = 0 \qquad E_1 = -i\omega\mu\psi_E$$
$$H_2 = \frac{\partial\psi_E}{\partial u_3} \qquad H_3 = -\frac{1}{1+\zeta(u_3)/d}\frac{\partial\psi_E}{\partial u_2} \tag{8}$$

where ψ_E and ψ_M are solutions of the scalar wave equation (3). The boundary conditions for ψ are determined by those for the fields at the planes ($u_2 = 0, d$). They are $E_3 = 0$ for the TM mode and $E_1 = 0$ for the TE mode (i.e., in the new coordinate system they are equivalent to the standard boundary conditions for

smooth waveguides). This allows us to solve (3a) and to substitute the value of k_2 into (3b), thus reducing the problem to a one-dimensional form. The allowed values of k_2 are

$$k_2 = (n\pi)/d, \qquad n \geq 1. \tag{9}$$

By examination of Maxwell's equations, in the new coordinate system it may be shown that there is a mode which becomes the TEM mode in the limit of zero roughness. The electric and magnetic fields associated with this mode are transverse to the u_3 direction and they do not depend on the u_2 coordinate, and can be considered a special case of the TM mode with $k_2 = 0$. The nonzero values of n correspond to higher modes of propagation.

Now that the problem has been reduced to one dimension, it is convenient to rewrite (6) as

$$V''(u) + \left[k^2 - \Omega(u)\right] V(u) = 0 \tag{10}$$

with

$$k^2 = k_0^2 - k_2^2, \qquad k_2^2 \neq 0 \tag{11}$$

where k_2 takes the values specified in (9) depending on the mode and $\Omega(u)$ is

$$\Omega(u) = \frac{\zeta''(u) - 4k_2^2\zeta(u)}{2d} + O\left(\left[\zeta/d\right]^2\right). \tag{12}$$

We will now examine the behavior of a wave fed into a section of rough waveguide; in particular we calculate the reflection coefficient of the section to "lowest order" in the roughness, ζ/d. To that end we impose boundary conditions of the form

$$V(u) \underset{u \to \infty}{\sim} te^{ik u} \tag{13a}$$

$$V(u) \underset{u \to -\infty}{\sim} e^{ik u} + re^{-ik u} \tag{13b}$$

where t and r are constants that we will now determine. The boundary conditions for $U(u)$ are identical, remembering that the roughness only has a finite extent.

The differential equation (10) with the boundary conditions (13a) and (13b) is equivalent to the following integral equation [8]:

$$V(u) = e^{ik u} + \frac{1}{2ik}\int_{-\infty}^{\infty} e^{ik|u-v|}\Omega(v)V(v)\,dv \tag{14}$$

where $e^{ik|u-v|}/(2ik)$ is the Green function for the waveguide with no roughness. The reflection coefficient is defined in terms of the Poynting vector, which is readily shown to be given by $|r|^2$ (using also the fact that $U(u)$ and $V(u)$ are identical outside the rough region). Since the roughness starts at $u = 0$, we may simplify the absolute value in the exponent in (14), and identify r, implying

$$R(k) = \frac{1}{4k^2}\left|\int_{-\infty}^{\infty} e^{ik v}\Omega(v)V(v)\,dv\right|^2. \tag{15}$$

We cannot solve (14) exactly and will resort to a perturbation treatment; the validity of this treatment relies on the reflection coefficient being small. At zeroth order $V^0(u) = e^{ik u}$ and we may substitute this into the integral in (14) to give the reflection coefficient in the first Born approximation (note that the propagation constant has not changed to this level of approximation):

$$R(k) \simeq \frac{k_0^4}{d^2k^2}\left|\hat{\zeta}(-2k)\right|^2 \tag{16}$$

where $\hat{\zeta}(q)$ is the Fourier transform of $\zeta(u)$. The significance of $-2k$ is that it is the momentum transfer required for reflection. This expression is quite general, not being specific to the randomly rough case.

For a periodically varying section of waveguide, the Fourier transform, $\hat{\zeta}(q)$, is proportional to the length of the section, L; so the power spectrum (and hence the reflection coefficient) are proportional to L^2. However, for a random section, the reflection coefficient *itself* is proportional to L [9]:

$$\langle R(k)\rangle \simeq \frac{k_0^4}{d^2k^2}\left\langle\left|\hat{\zeta}(-2k)\right|^2\right\rangle. \tag{17}$$

We will now determine the *average* reflection coefficient for a section of length L. To do this we must specify the statistical characteristics of the roughness: for concreteness, we choose the case of $\zeta(u)$ having Gaussian correlations and find

$$\langle R(k)\rangle \simeq \delta^2\frac{k_0^4}{d^2k^2}L\sqrt{\pi}\,\sigma e^{-\sigma^2k^2} \tag{18}$$

where δ is the root-mean-square roughness. The factors of L and δ^2 are quite general [9], as is the factor of the correlation length, σ, if the correlations have a characteristic length scale.

Let us note the marked difference of the reflection coefficient between the TE (or TM) and TEM modes as $k \to 0$: in the former case R diverges (since $k_0 \to k_2$, which is finite) and in the latter tends to zero. The origin of this distinction is that the TEM mode is increasingly (as $k \to 0$) insensitive to fluctuations in the width of the waveguide, whereas the TE mode retains its sensitivity, even as $k \to 0$. The reason for the divergence is that the perturbation, $\Delta(u)$, effectively is a uniform increase, or decrease, in width of extent L. In general this distinction may not matter in practical situations, as $k \not\to 0$. We cannot trust the Born approximation when R becomes comparable to unity.

An interesting feature that will arise at higher order is the enhancement of backscattering [10] by a factor of 2, due to the constructive interference between the time-reversed paths scattered more than once. Another effect only occurring at higher order is a change in the dispersion relation; this is unlike the "ridge" case, where the change occurs already at first order. The reason for this distinction is the lack of a predominant wave vector in the rough case, which leads to the splitting of the dispersion relation in the ridge case.

III. Conclusions

To conclude, we have utilized a transformation of coordinates to calculate a general expression for the reflection coefficient of a rough section of perfectly conducting waveguide. We find a dramatic difference between the TEM mode on the one hand and the TE and TM modes on the other.

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Spectral Estimation for the Transmission Line Matrix Method

JACK D. WILLS

Abstract—**Spectral estimation for the transmission line matrix (TLM) method by use of the discrete Fourier transform and fast Fourier transform is reviewed. Error bounds are given and checked by means of a numerical example. A new spectral estimation method based on Prony's method is presented for use with TLM. A numerical example shows that the new method allows an order of magnitude reduction in the number of iterations in the TLM method for equal accuracy.**

I. Introduction

The transmission line matrix (TLM) method for microwave circuit analysis calculates the time-domain variation of the electromagnetic fields in response to an arbitrarily chosen excitation [1]. Because of the discrete nature of the TLM method the output waveform is not a continuous function; it is a sequence of delta functions of varying amplitude. These delta functions are separated in time by Δt, which depends on the cell size used in the TLM model. This time is given by

$$\Delta t = \Delta l / c$$

where Δl is the spacing between nodes and c is the velocity of light in free space.

Frequently the desired information from TLM analysis is not the time-domain response but rather frequency-domain information. A common use of TLM is to determine the resonant frequencies of the characteristic modes of a microwave structure. To obtain these frequency-domain data we must apply some spectral estimation method to the time-domain output data. In the remainder of this paper we shall review spectral estimation methods presently in use for TLM, suggest an alternative spectral estimation method which appears promising, and compare numerical results for a typical problem.

II. Present Methods

In the original paper describing TLM, Johns and Beurle used Fourier transform techniques to obtain the frequency response of the circuit [2]. Specifically they applied the Fourier integral to the sequence of delta functions which represented the time-domain response of the circuit. By an application of the sifting property of the delta function, they expressed the frequency-domain response as a pair of finite summations:

$$\mathrm{Re}\left(F\left(\frac{\Delta}{\lambda}\right)\right) = \sum_{k=1}^{NI} I_k \cos\left(2\pi k \frac{\Delta l}{\lambda}\right) \tag{1}$$

$$\mathrm{Im}\left(F\left(\frac{\Delta l}{\lambda}\right)\right) = \sum_{k=1}^{NI} I_k \sin\left(2\pi k \frac{\Delta l}{\lambda}\right) \tag{2}$$

where $F(\Delta l/\lambda)$ is the frequency response, I_k is the output impulse response at time $t = k(\Delta l/c)$, and NI is the total number of iterations used in the TLM method.

We can then form either $|F(\Delta l/\lambda)|$ or $|F(\Delta l/\lambda)|^2$ (which are analogous to voltage magnitude or power respectively) and plot this as a function of frequency. Resonant frequencies of the microwave circuit correspond to peaks (local maxima) of the plot.

These equations are written in a normalized form. Johns and Beurle rescaled so as to make the interval between pulses unity rather than Δt. Thus they divided all times by Δt and multiplied all frequencies by Δt. We note that

$$\Delta t f = \frac{\Delta l}{c} f = \frac{\Delta l}{\lambda f} f = \frac{\Delta l}{\lambda} \tag{3}$$

which gives the frequency variable used in (1) and (2). By Nyquist's criterion, $f \leqslant 1/2\Delta t$, which ensures that $0 \leqslant \Delta l/\lambda \leqslant 0.5$.

There are two sources of error in determining resonant frequency in this manner. The first is a truncation error. One can only run the TLM simulation for a finite number of iterations, which we denote by NI. This has the effect of viewing the true (infinitely long) time-domain response through a rectangular window. The duration of this window is $NI\Delta t$. In the frequency domain, the effect of this windowing is to convolve the true frequency spectrum with the function

$$F_{\mathrm{window}}\left(\frac{\Delta l}{\lambda}\right) = \frac{\sin\left(\pi NI \Delta t \frac{\Delta l}{\lambda}\right)}{\pi NI \Delta t \frac{\Delta l}{\lambda}}. \tag{4}$$

The effects of this convolution are twofold. It widens the peaks in the frequency response plot, and the side lobes of the $\sin x/x$ function cause the side lobes due to one response peak to overlap the main lobe of another response peak. The result is that the observed local maxima of the frequency response are shifted away from their true values. Johns has derived an error bound for this shift [3]. For two response peaks of equal amplitude, separated by normalized frequency $S = \Delta l/\lambda$, Johns found the maximum truncation error $\Delta S_{\mathrm{trunc}}$ to be bounded by

$$\Delta S_{\mathrm{trunc}} \leqslant \pm \frac{3}{S(NI\pi)^2}. \tag{5}$$

An additional error source arises in finding the peaks in the response curve. While the frequency response given by the finite summation in (5) is a continuous function of frequency, we can only evaluate this equation at a finite number of points. If we evaluate the frequency response at NF points equally spaced across the interval (0, 0.5) any peak we find may be shifted from its true position by a normalized frequency of $\pm 1/4NF$. This

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